Dependence of the critical temperature on the Higgs field reparametrization

E. Di Grezia*

Università Statale di Bergamo, Facoltà di Ingegneria,

viale Marconi 5, I-24044 Dalmine (BG), Italy

and Istituto Nazionale di Fisica Nucleare,

Sezione di Milano, via Celoria 16, I-20133 Milan, Italy

S. Esposito†

Dipartimento di Scienze Fisiche, Università di Napoli "Federico II"

and Istituto Nazionale di Fisica Nucleare, Sezione di Napoli,

Complesso Universitario di Monte S. Angelo, via Cinthia, I-80126 Naples, Italy

G. Salesi‡

Università Statale di Bergamo, Facoltà di Ingegneria,

viale Marconi 5, I-24044 Dalmine (BG), Italy

and Istituto Nazionale di Fisica Nucleare,

Sezione di Milano, via Celoria 16, I-20133 Milan, Italy

Abstract

We show that, despite of the reparametrization symmetry of the Lagrangian describing the interaction between a scalar field and gauge vector bosons, the dynamics of the Higgs mechanism is really affected by the representation gauge chosen for the Higgs field. Actually, we find that, varying the parametrization for the two degrees of freedom of the complex scalar field, we obtain different expressions for the Higgs mass: in its turn this entails different expressions for the critical temperatures, ranging from zero to a maximum value, as well as different expressions for other basic thermodynamical quantities.

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The Higgs mechanism is the basic ingredient of many theories, ranging from superconductivity to elementary particle physics, where the phenomenon of spontaneous symmetry breaking (SSB) plays a key role. In fact, the original idea by Higgs (and others) [1, 2] to give a non-vanishing mass to gauge vector bosons through the coupling with a scalar field ϕ, in the framework of the Nambu model of SSB [3], was inspired by the fundamental works by Ginzburg and Landau [4] aimed at accounting the emergence of short-range electromagnetic interactions, mediated by massive-like photons, in superconductors and superfluids.

One of the topical issues in present day experimental research on elementary particles is, indeed, just the search for the Higgs boson at LHC [5], whose effective discovery will be the keystone for the complete confirmation of the Standard Model of electroweak interaction by Glashow, Weinberg and Salam [6]. Instead, in superconductivity mean-field theory (just to mention one application in condensed matter physics), the scalar field describes the dynamics of the Cooper pairs in the superconductors, ϕ being interpreted as the wave function of the Cooper pairs in their center-of-mass frame. For the sake of definitess, and without loss of generality, in this paper we shall often refer to superconductors. In the simplest version of the Ginzburg-Landau (GL) model (which will be considered here), the Lagrangian density describing a scalar field ϕ interacting with the electromagnetic field A_μ is given by (hereafter we assume $m^2 < 0$)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - m^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 \,, \tag{1}$$

where λ is the self-interaction coupling giving the strength of the Cooper pair binding, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength and $D_\mu = \partial_\mu + 2ieA_\mu$ is the covariant derivative for the Cooper pair with electric charge $2e$. The field ϕ is assumed to be a complex quantity, with *two* degrees of freedom, but only *one* non-vanishing vacuum expectation value (VEV), namely the condensation value of the field below a critical temperature T_c. Usually, such two degrees of freedom are represented by the real and imaginary part of ϕ (see, for example, ref. [2]), but other representations (for example, in terms of modulus and phase of ϕ) are of course possible. As is well known, the above Lagrangian is invariant under reparametrization of ϕ, and the various representations of the complex field describe. at the classical (tree) level, the same physical reality. This is true also in two (or more) Higgs models, where it holds the "rephasing" invariance of the Higgs multiplet (see refs.[7]). Already in 1984, however, some authors [8] pointed out that different representations of

ϕ can lead to *different* expressions for the critical temperature T_c where the condensation phenomenon takes place, since the dynamics ruled by different degrees of freedom could lead, in principle, to different predictions [9], when considering also the radiative, non-tree corrections. Nevertheless, it should be emphasized that such a difference is purely formal *if only one condensation occurs* since, in this case, the different dependence of T_c on the model parameters m^2, λ is not observable, being such parameters not directly measurable (they can be deduced just by measurement of T_c or other observables). The situation is, instead, just different if two (or more) condensations take place in peculiar physical system as, e.g., same non-standard superconductors. In this case, the difference between the critical temperatures leads to distinct superconductive phases, entailing two discontinuities in the specific heat and unusual magnetic properties. In a recent series of papers of ours [9, 10, 11] we have, indeed, studied in detail this possibility, and it is very remarkable that the two different representations mentioned above (real and imaginary parts versus modulus and phase) account, in a very simple manner, for the apparently exotic properties observed in the superconductivity of strontium ruthenate [11, 12, 13]. These observations evidently urge to consider accurately the problem of the field reparametrization in the Higgs mechanism: actually, in the present paper we shall expound a sufficiently general and comprehensive analysis of the physical implications of the field representation gauge. We start our analysis from the very general assumption that the complex scalar field $\phi(x)$ appearing in (1) is endowed with two different degrees of freedom described by the real scalar fields $a(x)$, $b(x)$. The only non-vanishing VEV of $\phi(x)$ is introduced by means of the (non-zero) real parameter a_0 as follow [15]

$$< \phi(x) > = \frac{a_0}{\sqrt{2}} \,. \tag{2}$$

Without loss of generality we assume that such VEV corresponds to $a = a_0$, $b = 0$, that is, the field $a(x)$ condenses for $a = a_0$, while the field $b(x)$ does not condense. Furthermore, we also assume that the field $\phi(x)$ can be expanded in a Taylor series around such values; on very general grounds, we can then represent the field $\phi(x)$ with the following formula:

$$\phi(x) = \sum_{n,m} a_0^{1-n-m} (c_{nm} + i d_{nm}) a^n b^m \,. \tag{3}$$

Here c_{nm}, d_{nm} are real, dimensionless coefficients that characterize the different representations, while the factor a_0^{1-n-m} is introduced for dimensional reasons (both the fields a, b have the same physical dimensions as a_0). Notice that, in order to preserve the appearance

of *two* degrees of freedom, at least one c-coefficient and one d-coefficient must be different from zero:

$$\exists n' \in \mathbb{N},\ \exists m' \in \mathbb{N}_0 :\ c_{n'm'} \neq 0,$$
$$\exists n'' \in \mathbb{N}_0,\ \exists m' \in \mathbb{N} :\ d_{n''m''} \neq 0\,. \tag{4}$$

The most "popular" parametrizations are the "Gauss representation"

$$\phi = \frac{1}{\sqrt{2}}(a + ib) \tag{5}$$

where we can observe a condensation of the real (or imaginary) part; or the "Euler representation"

$$\phi = \frac{1}{\sqrt{2}}\, a\, \mathrm{e}^{ib/a_0}\,, \tag{6}$$

where we can observe a condensation of the modulus (or phase). Correspondingly the c_{nm}, d_{nm} coefficients take the values

$$c_{nm} = \frac{1}{\sqrt{2}}\delta_{n1}\delta_{m0} \qquad\qquad d_{nm} = \frac{1}{\sqrt{2}}\delta_{n0}\delta_{m1} \tag{7}$$

and

$$c_{nm} = \frac{1}{\sqrt{2}}\frac{(-1)^k}{(2k)!}\delta_{n1}\delta_{m,2k} \qquad\qquad d_{nm} = \frac{1}{\sqrt{2}}\frac{(-1)^k}{(2k+1)!}\delta_{n1}\delta_{m,2k+1}\,, \tag{8}$$

respectively. In general, the coefficients c_{nm}, d_{nm} are not completely arbitrary, but have to satisfy several constraints, that will be discussed in the following. First of all, we take into account that only one non-vanishing VEV exists, given by Eq. (2). By equating Eqs. (2) and (3), for $a = a_0$, $b = 0$ we obtain:

$$\sum_n c_{n0} = \frac{1}{\sqrt{2}} \qquad\qquad \sum_n d_{n0} = 0\,. \tag{9}$$

Let us now proceed with the Higgs mechanism, by expanding ϕ in Eq. (3) around its VEV,

$$a \simeq a_0 + \tilde{a}\,, \qquad\qquad b \simeq \tilde{b} \tag{10}$$

($\tilde{a}$, $\tilde{b}$ are fluctuations fields), and inserting the resulting expression in Lagrangian (1) (we will consider only terms up to the second order in the fields). The kinetic terms for the fields $\tilde{a}$ and $\tilde{b}$ coming out from (1) are

$$\begin{aligned}
\mathcal{L}_{\mathrm{kin}} \ \simeq\ & \left[\left(\sum_n c_{n0}\right)^2 + \left(\sum_n n d_{n0}\right)^2\right] \partial_\mu\tilde{a}\,\partial^\mu\tilde{a} + \left[\left(\sum_n c_{n1}\right)^2 + \left(\sum_n d_{n1}\right)^2\right] \partial_\mu\tilde{b}\,\partial^\mu\tilde{b} \\
& +\ 2\left[\left(\sum_n c_{n1}\right)\left(\sum_n n c_{n0}\right)\left(\sum_n d_{n1}\right)\left(\sum_n n d_{n0}\right)\right] \partial_\mu\tilde{a}\,\partial^\mu\tilde{b}\,.
\end{aligned} \tag{11}$$

By diagonalizing the above Lagrangian we deduce the following constraints:

$$\begin{aligned}
&\left(\sum_n c_{n0}\right)^2 + \left(\sum_n n d_{n0}\right)^2 = \frac{1}{2} \\
&\left(\sum_n c_{n1}\right)^2 + \left(\sum_n d_{n1}\right)^2 = \frac{1}{2} \\
&\left(\sum_n c_{n1}\right)\left(\sum_n n c_{n0}\right) + \left(\sum_n d_{n1}\right)\left(\sum_n n d_{n0}\right) = 0\,.
\end{aligned} \tag{12}$$

The potential terms for the scalar field

$$\mathcal{L}_{\rm pot} = -m^2\phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^2\,, \tag{13}$$

can be analogously evaluated up to second order terms. By taking into account the constraints in Eqs. (9) and (12), we are able to write down the mass-potential terms in the following form

$$\begin{aligned}
\mathcal{L}_{\rm pot} \simeq\ & -\left[\frac{a_0^2}{2}\left(m^2+\frac{\lambda a_0^2}{8}\right)\right] - \left[2\frac{a_0}{\sqrt{2}}\left(m^2+\frac{\lambda a_0^2}{4}\right)\left(\sum_n n c_{n0}\right)\right]\tilde{a} \\
& -\left[2\frac{a_0}{\sqrt{2}}\left(m^2+\frac{\lambda a_0^2}{4}\right)\left(\sum_n c_{n1}\right)\right]\tilde{b} - \left[\frac{2}{\sqrt{2}}\left(m^2+\frac{\lambda a_0^2}{4}\right)\left(\sum_n n c_{n1}\right)\right. \\
& \left.+\lambda a_0^2\left(\sum_n n c_{n0}\right)\left(\sum_n c_{n1}\right)\right]\tilde{a}\tilde{b} - \left[\left(m^2+\frac{\lambda a_0^2}{4}\right)\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\sum_n n(n-1)c_{n0}\right)\right. \\
& \left.+\frac{\lambda a_0^2}{2}\left(\sum_n n c_{n0}\right)^2\right]\tilde{a}^2 - \left[\left(m^2+\frac{\lambda a_0^2}{4}\right)\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\sum_n c_{n2}\right)\right. \\
& \left.-\frac{\lambda a_0^2}{2}\left(\sum_n n c_{n0}\right)^2 + \frac{\lambda a_0^2}{4}\right]\tilde{b}^2\,.
\end{aligned} \tag{14}$$

For a generic representation of the field ϕ, the terms in $\tilde{a}$, $\tilde{b}$ and $\tilde{a}\tilde{b}$ do not vanish, so that, in this case, the mass eigenstates of the system are not the fields $\tilde{a}$, $\tilde{b}$ but a linear combination of them. However, as we will see below (see also [2]), the quantity relevant for the calculation of the critical temperature, or of other observables, is the trace of the squared mass matrix, that is invariant under the mentioned transformation. Therefore, for simplicity, we will limit our attention to the following part of the potential term:

$$\mathcal{L}_2 = -\frac{1}{2}m_a^2\tilde{a}^2 - \frac{1}{2}m_b^2\tilde{b}^2 \tag{15}$$

with

$$
\begin{aligned}
m_a^2 &= 2m^2\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\sum_n n(n-1)c_{n0}\right)+ \\
&\quad +\lambda a_0^2\left[\left(\sum_n nc_{n0}\right)^2+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\sum_n n(n-1)c_{n0}\right)\right], \qquad (16) \\
m_b^2 &= 2m^2\left(\frac{1}{2}+\frac{2}{\sqrt{2}}\sum_n c_{n2}\right)+\lambda a_0^2\left[\frac{1}{2}-\left(\sum_n nc_{n0}\right)^2+\frac{1}{2}\left(\frac{1}{2}+\frac{2}{\sqrt{2}}\sum_n c_{n2}\right)\right].
\end{aligned}
$$

We also introduce the following quantities that will directly enter in the expression for the critical temperature:

$$
\begin{aligned}
M_a^2 &\equiv m_a^2-m_a^2(a_0=0)=\lambda a_0^2\left[\left(\sum_n nc_{n0}\right)^2+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\sum_n n(n-1)c_{n0}\right)\right], \\
M_b^2 &\equiv m_b^2-m_b^2(a_0=0)=\lambda a_0^2\left[\frac{1}{2}-\left(\sum_n nc_{n0}\right)^2\frac{1}{2}\left(\frac{1}{2}+\frac{2}{\sqrt{2}}\sum_n c_{n2}\right)\right]. \qquad (17)
\end{aligned}
$$

By using the same notations as in Ref. [2], the Higgs "mass" is thus given by (of course, in M_H^2 the terms proportional to m^2 are not included, but only the corrections proportional to a_0^2 do appear)

$$
M_H^2 \;=\; \mathrm{Tr}\{M_s^2\}=M_a^2+M_b^2=\lambda a_0^2\left[1+\frac{1}{2\sqrt{2}}\sum_n\left(n(n-1)c_{n0}+2c_{n2}\right)\right] \qquad (18)
$$

and, evidently, *depends on the representation chosen* (through c_{n0}, c_{n2}). Instead, from the vector boson mass term

$$
\mathcal{L}_{M_V}=e^2|\phi|^2A_\mu A^\mu\simeq\frac{e^2a_0^2}{2}A_\mu A^\mu\,. \qquad (19)
$$

we obtain the photon mass from

$$
M_V^2=e^2a_0^2\,, \qquad (20)
$$

that is, obviously independent of the representation chosen. In order to get an expression for the critical temperature of the system, we have to consider the quantum temperature-dependent, radiative corrections to the scalar potential that, at tree level and in its minimum (2), is (see the first term in Eq. (14))

$$
U_0=\frac{1}{2}m^2a_0^2+\frac{1}{16}\lambda a_0^4\,. \qquad (21)
$$

For $e^4 \ll \lambda$ we can neglect $T = 0$ corrections to the potential; by assuming also that $T^2 \gg -m^2, \lambda a_0^2, e^2 a_0^2$, the T-dependent quantum correction term is given by [2]

$$U_1 \simeq -\frac{4\pi^2 T^4}{90} + \frac{T^2}{24}\left[M_H^2 + 3M_V^2\right] , \tag{22}$$

so that the total potential can be written as

$$U \simeq \frac{1}{2} m_{eff}^2 a_0^2 + \frac{1}{16}\lambda a_0^4 - \frac{4\pi^2 T^4}{90} , \tag{23}$$

with

$$m_{\text{eff}}^2 \simeq m^2 + \frac{T^2}{12 a_0^2}\left[M_H^2 + 3M_V^2\right] . \tag{24}$$

The critical temperature T_c of the system is, then, defined by the equation

$$m_{\text{eff}}^2(T_c) = 0 , \tag{25}$$

or

$$T_c^2 = \frac{-12 m^2}{\lambda H + 3e^2} , \tag{26}$$

with

$$H \equiv \frac{M_H^2}{\lambda a_0^2} = 1 + \frac{1}{2\sqrt{2}}\left[\sum_n \left(n(n-1)c_{n0} + 2c_{n2}\right)\right] . \tag{27}$$

The key result in Eq. (26) clearly shows that *the observable T_c does depend on the representation chosen for the field ϕ* through the H term (or, what is the same, through the Higgs mass). As already pointed out above, however, such a dependence is only formal if the system possesses only one critical temperature, since the parameters λ and m^2 appearing in the lagrangian (1) are not directly observable. This is not the case, instead, for systems showing more than one critical temperature, as extensively discussed in [9, 10], so that it is quite relevant to discuss the possible consequences of Eq. (26). First of all, let us observe that, for the standard representations (5) and (6), we recover the known results [2, 8] namely

$$T_c^2 = \frac{-12 m^2}{\lambda + 3e^2} , \tag{28}$$

and

$$T_c^2 = \frac{-16 m^2}{\lambda + 4e^2} , \tag{29}$$

corresponding to Higgs masses $M_H^2 = \lambda a_0^2$ and $M_H^2 = \frac{3}{4}\lambda a_0^2$, respectively. It is noticeable that the above two results (and other similar values [14]; cf. also [8] for the Glashow-Weinberg-Salam model) can be here directly obtained as particular cases of a general theory, while

in literature they come out from different and very elaborate theoretical approaches. The latter value, for example, was derived using the real-time Green-function approach. In general, because of Eq. (27), the H term parameterizes the relative strength between the self-interaction of the Cooper pairs (ruled by λ) and the electromagnetic interaction (ruled by e). It is quite remarkable that such parameter, and thus T_c, depends on only "two" coefficients, c_{n0} and c_{n2} (for all n). This fact leads to relevant consequences. Firstly, since the d_{nm} coefficients do not contribute to the expression of T_c (or M_H), representations of ϕ that differ only for the imaginary part $\mathrm{Im}\{\phi\}$ give the same T_c (and M_H). However, this fact does not at all imply that we can consider just real representations of ϕ (contrary to the assumption of *two*, not one, degrees of freedom): in fact, from the constraints (9), (12) it immediately follows that not all d_{nm} coefficients can vanish. We have, then, a further limitation since, from Eq. (27), it is evident that T_c (and M_H) depends only on the coefficients c_{n0}, c_{n2}. This means that only the terms a^n and $a^n b^2$ in the expansions of ϕ contribute to T_c (and M_H), so that representations of ϕ whose real parts differ in their expansion around the VEV ($a = a_0$, $b = 0$) only for odd power terms in b or for $O(b^4)$ even power terms give the same T_c (and M_H). Another constraint on the coefficients c_{n0}, c_{n2} (which can assume, of course, even negative values) comes from Eq. (18) by requiring that the Higgs mass squared (after the condensation) is a positive quantity

$$\sum_n \left(n(n-1) c_{n0} + 2 c_{n2} \right) \geq -2\sqrt{2} \, . \tag{30}$$

Provided that such condition is satisfied, from Eq. (26) we obtain the important result that, changing the representation of ϕ, the critical temperature of the system cannot assume any arbitrarily large value, but is bounded in the interval

$$0 \leq T_c \leq T_c^{\max} \, , \tag{31}$$

with

$$T_c^{\max} = 2 \sqrt{\frac{-m^2}{e^2}} \, , \tag{32}$$

corresponding to the (reverse of the) interval $M_H \in [0, \infty)$. Eq. (26) can, then, be rewritten in the expressive form:

$$T_c = T_c^{\max} \sqrt{\frac{1}{1 + \frac{\lambda H}{3 e^2}}} \, , \tag{33}$$

and the dependence of T_c on the "representation factor" H is depicted in Fig. 1.

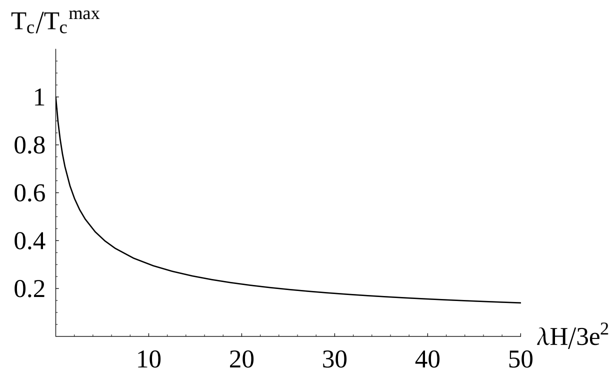

FIG. 1: Critical temperature versus the "representation factor" H.

Note that $T_c \to 0$ (that is, no superconductivity) for $M_H \to \infty$ ($H \to \infty$) or $\lambda/e^2 \to \infty$, that is when the Cooper pair self-interaction is much stronger than the electromagnetic interaction among electrons. Instead, the critical temperature approaches its maximum value in (32) when the Higgs mass tends to zero ($H \to 0$) or $\lambda/e^2 \to 0$, that is for a Cooper pair self-interaction much weaker in comparison to the electromagnetic interaction. A possible example of representation of ϕ that, independently of the value of λ/e^2, realizes such a case (that is, for which $M_H = 0$) is the following:

$$\phi = \frac{a}{\sqrt{2}}\left(3 - 2\frac{a}{a_0}\right) + i\frac{b}{\sqrt{2}}\left(1 - 2\frac{b}{a_0}\right) \tag{34}$$

[but, of course, many other representations are possible, according to what discussed above, provided that the constraints (9), (12) and (26) are satisfied]. Interestingly enough, the maximum critical temperature in (32) has an impressive physical interpretation in terms of the entropy of the system. In fact, from the standard expression of the free energy F in the GL model (evaluated in the VEV of the Higgs field) [10],

$$F = F_0 + \frac{1}{2}a(T)a_0^2 + \frac{\lambda}{16}a_0^4 \tag{35}$$

with

$$a(T) = -m^2\left(1 - \frac{T^2}{T_c^2}\right), \tag{36}$$

the entropy S of the system can be easily computed from

$$S = -\frac{\partial F}{\partial T}, \tag{37}$$

obtaining:

$$S = \frac{a_0^2}{12}(\lambda H + 3e^2)T = \left(\frac{M_H^2}{12} + \frac{M_V^2}{4}\right)T\,. \tag{38}$$

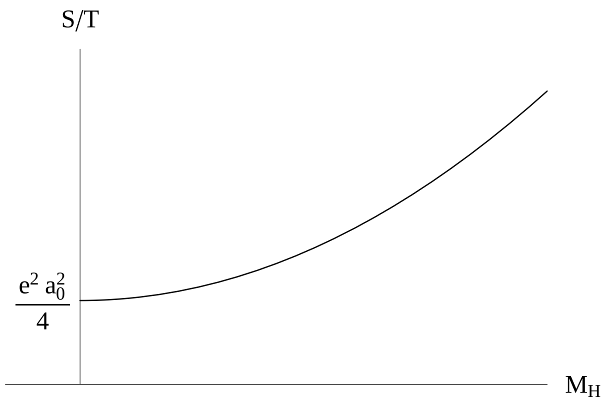

FIG. 2: Entropy versus the Higgs mass, for a given temperature.

For a given temperature T (see Fig.2), the entropy increases for increasing Higgs mass, starting from a minimum for $M_H = 0$. Thus, the maximum critical temperature in (32) corresponds, for given temperature T, to the minimum of the entropy of the system (different from zero for a non-vanishing v.e.v of ϕ) or, what is the same, to the maximum possible order of the system. This is what expected, since an higher temperature corresponds to a smaller Higgs mass, which in its turn advantages the transition to the more ordered broken phase.

Summing up, we have discussed how different reparametrizations of the scalar field ruling the Higgs mechanism (with two degrees of freedom and one non-vanishing VEV), as described by Eq. (3), affect the expression of the critical temperature of the system or, through the free energy (35), all the thermodynamical quantities of the standard GL models (applied, e.g., to superconductors and superfluids). This study is relevant only for physical systems that exhibit more than one critical temperatures, as the case, for example, of the superconductivity of strontium ruthenate [11]. Changing the possible representation of the scalar fields ϕ results (with some interesting exceptions, discussed above in detail) in different values for the Higgs mass and, through this parameter, in different critical temperatures. However, while the Higgs mass is, in general, not bounded (ranging from 0 to infinity), the critical temperatures of the system can increase from zero (that is no superconductivity) up to a maximum value $T_c^{\max}$, corresponding to the non-zero minimum of the entropy system (for given temperature). One possible representation for ϕ realizing such a limiting case is shown in Eq. (34), and can be regarded as a generalization of the standard representation (5) when highest order terms in a, b are included.

Although we have devoted our attention namely to the standard GL model which usually applies in condensed matter and solid state physics, nevertheless we expect that the consequences of our study will be not limited to that area of physics. In particular, further studies on the elementary particle physics sector, with special reference to the unification of the fundamental forces and to the phase transitions in the Early Universe, can disclose interesting new phenomena affecting our understanding of the cosmological evolution.

* Electronic address: digrezia@na.infn.it

† Electronic address: salvatore.esposito@na.infn.it

‡ Electronic address: salesi@unibg.it

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